

Common Core State Standards Mathematics



Student:
Teacher:

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.	3. Construct viable arguments and critique the reasoning of others.	5. Use appropriate tools strategically.	7. Look for and make use of structure.
2. Reason abstractly and quantitatively	4. Model with mathematics.	6. Attend to precision.	8. Look for and express regularity in repeated reasoning.

Grade 4 Mathematics Standards	Attempt	Attempt	Attempt	Pass
<p><u>Critical Areas</u> In Grade 4, instructional time should focus on four critical areas:</p> <ol style="list-style-type: none"> 1. Developing understanding and fluency with multiplication and developing understanding of dividing to find quotients involving multi-digit dividends. 2. Developing understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers. 3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. 				
4.OA Operations and Algebraic Thinking				
Use the four operations with whole numbers to solve problems.				
4.OA.1 Interpret a multiplication equation as a comparison (e.g., Interpret $35=5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.				
4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem,				

distinguishing multiplicative comparison from additive comparison (See Glossary, Table 2)).				
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.				
Gain familiarity with factors and multiples.				
4.OA.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.				
Generate and analyze patterns.				
4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example: Given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.				
4.NBT Number and Operations in Base Ten (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)				
Generalize place value understanding for multi-digit whole numbers.				
4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example: Recognize that $700 \div 70 = 10$ by applying concepts of place value and division.				
4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.				
4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.				
Use place value understanding and properties of operations to perform multi-digit arithmetic.				
4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.				
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.				

<p>4.NBT.6 Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>				
<p>4.NF Number and Operations-Fractions</p>				
<p>Extend understanding of fraction equivalence and ordering.</p>				
<p>4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(nxa)/(nxb)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>				
<p>4.NF.2 Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions (e.g., by using a visual fraction model).</p>				
<p>Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</p>				
<p>4.NF.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.</p>				
<p>4.NF.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p>				
<p>4.NF.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions (e.g., by using a visual fraction model: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2 \frac{1}{8} = 1 + \frac{1}{8} = \frac{8}{8} + \frac{1}{8}$).</p>				
<p>4.NF.3.c Add and subtract mixed numbers with like denominators (e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction).</p>				
<p>4.NF.3.d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).</p>				
<p>4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p>				
<p>4.NF.4.a Understand a fraction a/b as a multiple of $1/b$. For example: Use a visual fraction model to represent $\frac{5}{4}$ as the product of $5 \times (\frac{1}{4})$, recording the conclusion by the equations $\frac{5}{4} = 5 \times (\frac{1}{4})$.</p>				

<p>4.NF.4.b Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. For example: Use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$)</p>				
<p>4.NF.4.c Solve word problems involving multiplication of a fraction by a whole number (e.g., by using visual fraction models and equations to represent the problem). For example: If each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</p>				
<p>Understand decimal notation for fractions, and compare decimal fractions.</p>				
<p>4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100 (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade). For example: Express $3/10$ as $30/100$, and add $3/10 + 4/10 = 34/100$.</p>				
<p>4.NF.6 Use decimal notation for fractions with denominators 10 and 100. For example: Rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</p>				
<p>4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions (e.g., by using a visual model).</p>				
<p>4.MD Measurement and Data</p>				
<p>Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p>				
<p>4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example: Know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),</p>				
<p>4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p>				

<p>4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example: Find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</p>				
<p>Represent and interpret data.</p>				
<p>4.MD.4 make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example: From a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</p>				
<p>Geometric measurement: Understand concepts of angle and measure angles.</p>				
<p>4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.</p>				
<p>4.MD.5.a An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one degree angle,” and can be used to measure angles.</p>				
<p>4.MD.5.b An angle that turns through n one-degree angles is said to have an angle measure of n degrees.</p>				
<p>4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p>				
<p>4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle).</p>				
<p>4.G Geometry</p>				
<p>Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</p>				
<p>4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p>				
<p>4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</p>				
<p>4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures</p>				

and draw lines of symmetry.

TABLE 2. Common multiplication and division situations.⁷

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, ⁴ Area ⁵	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

⁴The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

⁵Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.¹

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction $\frac{A}{B}$ where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See *also*: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See *also*: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, $643 = 600 + 40 + 3$.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M , the first quartile is the median of the data values less than M . Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.² See *also*: median, third quartile, interquartile range.

Fraction. A number expressible in the form $\frac{a}{b}$ where a is a whole number and b is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) See *also*: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

¹Adapted from Wisconsin Department of Public Instruction, <http://dpi.wi.gov/standards/math/als.html>, accessed March 2, 2010.

²Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

Integer. A number expressible in the form a or $-a$ for some whole number a .

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is $15 - 6 = 9$. See *also*: first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.³

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.⁴ Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \div \frac{4}{3} = \frac{4}{3} \div \frac{3}{4} = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $\frac{5}{50} = 10\%$ per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See *also*: uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form $\frac{a}{b}$ or $-\frac{a}{b}$ for some fraction $\frac{a}{b}$. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

³Adapted from Wisconsin Department of Public Instruction, *op. cit.*

⁴To be more precise, this defines the *arithmetic mean*.

one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.⁵

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M , the third quartile is the median of the data values greater than M . Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ...