

# Common Core State Standards Mathematics



<b>Student:</b>
<b>Teacher:</b>

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.	3. Construct viable arguments and critique the reasoning of others.	5. Use appropriate tools strategically.	7. Look for and make use of structure.
2. Reason abstractly and quantitatively	4. Model with mathematics.	6. Attend to precision.	8. Look for and express regularity in repeated reasoning.

<b>Grade 3 Mathematics Standards</b>	Attempt	Attempt	Attempt	Pass
<b><u>Critical Areas</u></b> In Grade 3, instructional time should focus on four critical areas: 1. Developing understanding of multiplication and division and strategies for multiplication and division within 100. 2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1) 3. Developing understanding of the structure of rectangular arrays and of area 4. Describing and analyzing two-dimensional shapes.				
<b>3.OA Operations and Algebraic Thinking</b>				
<b>Represent and solve problems involving multiplication and division.</b>				
<b>3.OA.1</b> Interpret products of whole numbers, (e.g., $5 \times 7$ as a total number of objects in 5 groups of 7 objects each). For example: Describe a context in which a total number of objects can be expressed as $5 \times 7$ .				
<b>3.OA.2</b> Interpret whole-number quotients of whole numbers (e.g., Interpret $56 \div 8$ as the number of objects in each share of 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned equal shares of 8 objects each. For example: Describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$ ).				

<p><b>3.OA.3</b> Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem). See Table 2 attached.</p>				
<p><b>3.OA.4</b> Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example: Determine the unknown number that makes the equation true in each of the equations <math>8 \times ? = 48</math>, <math>5 = \quad \div 3</math>, <math>6 \times 6 = ?</math>.</p>				
<p><b>Understand properties of multiplication and the relationship between multiplication and division.</b></p>				
<p><b>3.OA.5</b> Apply properties of operations as strategies to multiply and divide (See Glossary, Table 2). Examples: If <math>6 \times 4 = 24</math> is known, then <math>4 \times 6 = 24</math> is also known. (Commutative Property of Multiplication) <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math>, then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math>, then <math>3 \times 10 = 30</math>. (Associative Property of Multiplication) Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math>. (Distributive Property). Students need not use formal terms for these properties.</p>				
<p><b>3.OA.6</b> Understand division as an unknown-factor problem. For example: Find <math>32 \div 8</math> by finding the number that makes 32 when multiplied by 8.</p>				
<p><b>Multiply and divide within 100.</b></p>				
<p><b>3.OA.7</b> Fluently multiply and divide within 100 using strategies such as the relationship between multiplication and division (e.g., Knowing that <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</p>				
<p><b>Solve problems involving the four operations, and identify and explain patterns in arithmetic</b></p>				
<p><b>3.OA.8</b> Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order-Order of Operations).</p>				
<p><b>3.OA.9</b> Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example: Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.</p>				
<p><b>3.NBT Number and Operations in Base Ten</b></p>				
<p><b>Use place value understanding and properties of operations to perform multi-digit arithmetic (A range of algorithms may be used).</b></p>				

<b>3.NBT.1</b> Use place value understanding to round whole numbers to the nearest 10 or 100.				
<b>3.NBT.2</b> Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.				
<b>3.NBT.3</b> Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., $9 \times 80$ , $5 \times 60$ ) using strategies based on place value and properties of operations.				
<b>3.NF</b> Number and Operations-Fractions (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8)				
<b>Develop understanding of fractions as numbers.</b>				
<b>3.NF.1</b> Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$ .				
<b>3.NF.2</b> Understand a fraction as a number on the number line; represent fractions on a number line diagram.				
<b>3.NF.2.a</b> Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.				
<b>3.NF.2.b</b> Represent a fraction $a/b$ on a number line diagram by marking off $a$ lengths $1/b$ from 0. Recognize that the resulting interval has size $a/b$ and that its endpoint locates the number $a/b$ on the number line.				
<b>3.NF.3</b> Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.				
<b>3.NF.3.a</b> Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.				
<b>3.NF.3.b</b> Recognize and generate simple equivalent fractions (e.g., $1/2 = 2/4$ ; $4/6 = 2/3$ ). Explain why the fractions are equivalent (e.g., by using a visual fraction model).				
<b>3.NF.3.c</b> Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. (e.g., Express 3 in the form $3 = 3/1$ ; recognize that $6/1 = 6$ ; locate $4/4$ and 1 at the same point of a number line diagram).				
<b>3.NF.3.d</b> Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$ , $=$ , or $<$ , and justify the conclusions (e.g., by using a visual fraction model).				
<b>3.MD</b> Measurement and Data				

<b>Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.</b>				
<b>3.MD.1</b> Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes (e.g., by representing the problem on a number line diagram).				
<b>3.MD.2</b> Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as $\text{cm}^3$ and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units (e.g., by using drawings such as a beaker with a measurement scale to represent the problem). (Excludes multiplicative comparison problems [problems involving notions of “times as much”; see Glossary, Table 2])				
<b>Represent and interpret data.</b>				
<b>3.MD.3</b> Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.				
<b>3.MD.4</b> Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters.				
<b>Geometric measurement: Understand concepts of area and relate area to multiplication and to addition.</b>				
<b>3.MD.5</b> Recognize area as an attribute of plane figures and understand concepts of area measurement.				
<b>3.MD.5.a</b> A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.				
<b>3.MD.5.b</b> A plane figure which can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units.				
<b>3.MD.6</b> Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).				
<b>3.MD.7</b> Relate area to the operations of multiplication and addition.				
<b>3.MD.7.a</b> Find the area of a rectangle with whole-number side lengths by tiling it, and showing that the area is the same as would be found by multiplying the side lengths.				
<b>3.MD.7.b</b> Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole –number products as rectangular areas in mathematical				

reasoning.				
<b>3.MD.7.c</b> Use tiling to show in a concrete case that the area of a rectangle with whole-number lengths $a$ and $b+c$ is the sum of $axb$ and $axc$ . Use area models to represent the distributive property in mathematical reasoning.				
<b>3.MD.7.d</b> Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping parts, applying this technique to solve real world problems.				
<b>Geometric measurement: Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.</b>				
<b>3.MD.8</b> Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.				
<b>3.G Geometry</b>				
<b>Reason with shapes and their attributes.</b>				
<b>3.G.1</b> Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.				
<b>3.G.2</b> Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example: partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.				

TABLE 2. Common multiplication and division situations.<sup>7</sup>

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$ , and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$ , and $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays, <sup>4</sup> Area <sup>5</sup>	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$ , and $p \div a = ?$	$? \times b = p$ , and $p \div b = ?$

<sup>4</sup>The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>5</sup>Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

## Glossary

**Addition and subtraction within 5, 10, 20, 100, or 1000.** Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example:  $8 + 2 = 10$  is an addition within 10,  $14 - 5 = 9$  is a subtraction within 20, and  $55 - 18 = 37$  is a subtraction within 100.

**Additive inverses.** Two numbers whose sum is 0 are additive inverses of one another. Example:  $\frac{3}{4}$  and  $-\frac{3}{4}$  are additive inverses of one another because  $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$ .

**Associative property of addition.** See Table 3 in this Glossary.

**Associative property of multiplication.** See Table 3 in this Glossary.

**Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

**Box plot.** A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.<sup>1</sup>

**Commutative property.** See Table 3 in this Glossary.

**Complex fraction.** A fraction  $\frac{A}{B}$  where  $A$  and/or  $B$  are fractions ( $B$  nonzero).

**Computation algorithm.** A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See *also*: computation strategy.

**Computation strategy.** Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See *also*: computation algorithm.

**Congruent.** Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

**Counting on.** A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

**Dot plot.** See: line plot.

**Dilation.** A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

**Expanded form.** A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example,  $643 = 600 + 40 + 3$ .

**Expected value.** For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

**First quartile.** For a data set with median  $M$ , the first quartile is the median of the data values less than  $M$ . Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.<sup>2</sup> See *also*: median, third quartile, interquartile range.

**Fraction.** A number expressible in the form  $\frac{a}{b}$  where  $a$  is a whole number and  $b$  is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) See *also*: rational number.

**Identity property of 0.** See Table 3 in this Glossary.

**Independently combined probability models.** Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

<sup>1</sup>Adapted from Wisconsin Department of Public Instruction, <http://dpi.wi.gov/standards/math/als.html>, accessed March 2, 2010.

<sup>2</sup>Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., “Quartiles in Elementary Statistics,” *Journal of Statistics Education* Volume 14, Number 3 (2006).

**Integer.** A number expressible in the form  $a$  or  $-a$  for some whole number  $a$ .

**Interquartile Range.** A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is  $15 - 6 = 9$ . See *also*: first quartile, third quartile.

**Line plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.<sup>3</sup>

**Mean.** A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.<sup>4</sup> Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

**Mean absolute deviation.** A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

**Median.** A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

**Midline.** In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

**Multiplication and division within 100.** Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example:  $72 \div 8 = 9$ .

**Multiplicative inverses.** Two numbers whose product is 1 are multiplicative inverses of one another. Example:  $\frac{3}{4}$  and  $\frac{4}{3}$  are multiplicative inverses of one another because  $\frac{3}{4} \div \frac{4}{3} = \frac{4}{3} \div \frac{3}{4} = 1$ .

**Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

**Percent rate of change.** A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by  $\frac{5}{50} = 10\%$  per year.

**Probability distribution.** The set of possible values of a random variable with a probability assigned to each.

**Properties of operations.** See Table 3 in this Glossary.

**Properties of equality.** See Table 4 in this Glossary.

**Properties of inequality.** See Table 5 in this Glossary.

**Properties of operations.** See Table 3 in this Glossary.

**Probability.** A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

**Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See *also*: uniform probability model.

**Random variable.** An assignment of a numerical value to each outcome in a sample space.

**Rational expression.** A quotient of two polynomials with a non-zero denominator.

**Rational number.** A number expressible in the form  $\frac{a}{b}$  or  $-\frac{a}{b}$  for some fraction  $\frac{a}{b}$ . The rational numbers include the integers.

**Rectilinear figure.** A polygon all angles of which are right angles.

**Rigid motion.** A transformation of points in space consisting of a sequence of

<sup>3</sup>Adapted from Wisconsin Department of Public Instruction, *op. cit.*

<sup>4</sup>To be more precise, this defines the *arithmetic mean*.

one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

**Repeating decimal.** The decimal form of a rational number. *See also:* terminating decimal.

**Sample space.** In a probability model for a random process, a list of the individual outcomes that are to be considered.

**Scatter plot.** A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.<sup>5</sup>

**Similarity transformation.** A rigid motion followed by a dilation.

**Tape diagram.** A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

**Terminating decimal.** A decimal is called terminating if its repeating digit is 0.

**Third quartile.** For a data set with median  $M$ , the third quartile is the median of the data values greater than  $M$ . Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

**Transitivity principle for indirect measurement.** If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

**Uniform probability model.** A probability model which assigns equal probability to all outcomes. *See also:* probability model.

**Vector.** A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

**Visual fraction model.** A tape diagram, number line diagram, or area model.

**Whole numbers.** The numbers 0, 1, 2, 3, ....